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# Quantum Groups

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## FOREWORD

This is the first volume of the Proceedings of the Euler International Mathematical Institute in St.Petersburg. Springer-Verlag has kindly agreed to incorporate these Proceedings in the format of the Lecture Notes in Mathematics.

Due to a variety of circumstances the first Semester of the Institute, in the fall of 1990, was not fully fledged. Instead of a combined research effort it was organized in the form of short workshops. So this volume contains the results of individual authors, brought to the attention of the participants, rather than the outcome of joint work carried out within the walls of the Institute. This will be changed as soon as the Institute begins to work in full strength.

I hope that the results obtained at the Euler Institute will interest the international mathematical community, so that our publications will become an important part of the Lecture Notes in Mathematics. I would like to thank Springer-Verlag for their invaluable help and good will.

Director of **EIMI**

L. Faddeev

## PREFACE

The Euler International Mathematical Institute in Leningrad was founded with the aim of enhancing contacts of Soviet and foreign mathematicians and to create opportunities for them to meet during two or three month periods to discuss current problems and carry out research within the mathematical topic chosen for this period.

The first semester (Fall 1990) was devoted to quantum groups, in view of the growing interest to this subject manifested by mathematicians of various specializations. Organizational questions combined with the desire to bring together as many participants as possible determined the structure of the first semester which was divided into three workshops:

1. Quantum groups, deformation theory and representation theory (15–28 October 1990);
2. Quantum groups, symmetries of dynamical systems and conformal field theory (12–25 November 1990);
3. Quantum groups, low-dimensional topology and link invariants (2–15 December 1990).

Each workshop was attended by approximately 15 Soviet mathematicians and 15 colleagues from Europe, Japan and the U.S.. All participants gave one-hour talks most of which are included in these Proceedings. According to the topical division, the contributions are grouped into three sections whose titles coincide with the titles of the workshops. The talks were delivered in the mornings, and in the afternoons some informal seminars were devoted mainly to detailed discussions of the problems mentioned in the talks. In particular, during the first workshop there was a seminar on unsolved problems in quantum groups. Some of these problems are published in Chapter 4 of these Proceedings. Many interesting problems are enounced also in the contributed papers; indeed one of the papers carries the title “On some unsolved problems in quantum group theory”.

From the topics of the workshops and the titles of the contributions it is clear that quantum groups and quantum Lie algebras draw the attention of mathematicians from quite different fields, including topology, algebra and analysis. These concepts are also actively exploited in modern theoretical physics. It may be interesting to note that almost all conferences on mathematical and theoretical physics, from high energy to solid state physics, planned for this summer (1991) include a section on quantum groups.

The organizers of the first semester of the Euler International Mathematical Institute hope that the contributions of actively working experts in quantum groups collected in these Proceedings will influence the rapid development of this field.

Leningrad  
September 1991

P.P.Kulish

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