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Non-Abelian Supercurrents And de Sitter Ground State In Electroweak Theory

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Abstract: We show that gauge symmetry breaking in the Weinberg-Salam model can be implemented by a mere change of variables and without any explicit gauge fixing. The change of variables entails the concept of supercurrent which has been widely employed in the study of superconductivity. It also introduces a separation between the isospin and the hypercharge, suggesting that our new variables describe a strongly coupled regime of the electroweak theory. We discuss the description of various embedded topological defects in terms of these variables. We also propose that in terms of our variables the Weinberg-Salam model can be interpreted in terms of a gravity theory with the modulus of Higgs field as dilaton and the de Sitter space as the ground state.

Keywords: Gauge Symmetry, Spontaneous Symmetry Breaking, Nonperturbative Effects.

Contents

1. Introduction

Experiments with the LHC accelerator at CERN may soon reveal the mechanism of the electroweak symmetry breaking, pivotal to our understanding of fundamental interactions. The symmetry breaking is supposed to proceed in a textbook manner: The Higgs field develops a constant ground state expectation value which gives a mass for the charged $W^{\pm}$ and the neutral Z bosons but leaves the photon massless.

However, there remains theoretical issues to be addressed. Among them, a theorem by Elitzur [1] states that it should be impossible to spontaneously break a local symmetry such as the electroweak gauge group $G_{\mathrm{WS}} = SU_L(2) \times U_Y(1)$. According to this theorem only global symmetries can be spontaneously broken. A gauge fixed theory avoids this conundrum since the local gauge symmetry becomes explicitly broken by the gauge fixing condition. Furthermore, both numerical lattice simulations [2] and formal arguments [3] show that the transition between the symmetric and the Higgs phase can proceed in an analytic manner along a continuous path in the phase diagram. In particular, since the gauge symmetry is unbroken in the symmetric phase it must remain unbroken also in the Higgs phase, and this appears to bring the Higgs mechanism of the Weinberg-Salam model in line with Elitzur's theorem.

Here we address these issues from a new perspective by showing how the entire electroweak Lagrangian can be written in terms of manifestly $SU_L(2) \times U_Y(1)$ invariant variables that are the analogs of the Meißner supercurrent in the Ginzburg-Landau approach to BCS superconductor. These variables can be interpreted in terms of spin-charge separation, in line with the spin-charge separation that has been previously employed in the context of strongly correlated electron systems in condensed matter physics [4], [5], [6]. Thus the proper interpretation of our variables appears to be in terms of the strongly coupled (strongly correlated) dynamics of electroweak theory. Furthermore, since the non-Abelian supercurrents implement the effects of gauge symmetry breaking by a mere change of variables and without any gauge fixing, any issues with Elitzur's theorem become obsolete.

We also show that the isospin-hypercharge separated electroweak Lagrangian can be given a gravitational interpretation in terms of conformal geometry. This suggests a dual relation between the strongly coupled electroweak theory and a theory of gravitation [7]. In particular, we propose that the ground state of the electroweak theory is the four dimensional de Sitter space, with the modulus of the Higgs field as the dilaton.

2. Abelian Higgs Model

We start by illustrating our proposal by considering a complex scalar field ϕ and a vector field A_i in three space dimensions, in the context of the conventional Landau-Ginzburg approach to BCS superconductivity. There are a total of five independent fields. We introduce an invertible change of variables to a set of five independent fields (J_i, ρ, θ),

$$\begin{array}{lcl} \phi & \rightarrow & \rho \cdot e^{i\theta} \\ A_i & \rightarrow & J_i = \frac{i}{4e|\phi|^2}\left[\phi^*(\partial_i - 2ieA_i)\phi - c.c.\right] \end{array} \tag{2.1}$$

Note that we have not yet detailed any physical model where these variables appear as field degrees of freedom. We now proceed to the Landau-Ginzburg Hamiltonian which is relevant to BCS superconductivity, with ϕ the scalar field that describes Cooper pairing of electrons and A_i the (Maxwellian) $U(1)$ magnetic vector potential,

$$\mathcal{H} = \frac{1}{2}B_i^2 + |(\partial_i - 2ieA_i)\phi|^2 + \lambda\left(|\phi|^2 - v^2\right)^2 . \tag{2.2}$$

Here B_i denotes the magnetic field. This Hamiltonian displays the familiar Maxwellian $U(1)$ gauge invariance

$$\begin{array}{lcl} \phi & \rightarrow & e^{2ie\eta}\phi \\ A_i & \rightarrow & A_i + \partial_i\eta \end{array}$$

In terms of the new fields (2.1) the Hamiltonian (2.2) is

$$\mathcal{H} = \frac{1}{4}\left(J_{ij} + \frac{\pi}{e}\widetilde{\sigma}_{ij}\right)^2 + (\partial_i\rho)^2 + \rho^2 J_i^2 + \lambda\left(\rho^2 - \eta^2\right)^2, \tag{2.3}$$

with

$$J_{ij} = \partial_i J_j - \partial_j J_i$$

and

$$\widetilde{\sigma}_{ij} = \epsilon_{ijk}\sigma_k = \frac{1}{2\pi}\left[\partial_i, \partial_j\right]\theta \tag{2.4}$$

Here σ_i is the string current, its support in $\mathbb{R}^3$ coincides with the worldsheet of the core of a nonrelativistic Abrikosov vortex. When (2.3) describes such a vortex, (2.4) subtracts a singular contribution that emerges from J_{ij}. This singularity also emanates in the third term in the *r.h.s.* of (2.3). There it becomes removed by the density ρ which vanishes on the worldsheet of the vortex core.

The Hamiltonian (2.3) involves only variables that are manifestly $U(1)$ gauge invariant, there is no local gauge invariance present in (2.3). But no gauge has been fixed in deriving (2.3) from (2.2). Instead, all gauge dependent quantities have been explicitly eliminated by the change of variables. Notice that Eq. (2.4) is invariant under a $U(1)$ gauge transformation that entails a shift in θ by a twice differentiable scalar function. Since (2.4) displays no gauge invariances, there are no issues with Elitzur's theorem. Moreover, since $\rho \geq 0$ there are no gauge invariant global symmetries to be spontaneously broken by the potential term even though the Meißner effect does reflect the properties of the potential term.

3. Non-Abelian Supercurrents

We wish to generalize the previous approach to the (bosonic sector of the) standard electroweak theory, defined by the classical Lagrangian

$$\mathcal{L}_{\rm WS} = \frac{1}{4}\vec{G}^2_{\mu\nu}(W) + \frac{1}{4}F^2_{\mu\nu}(Y) + |D_\mu\Phi|^2 + \lambda|\Phi|^4 + \mu^2|\Phi|^2 \tag{3.1}$$

We use the notation of [8]. For the moment we work in a spacetime with Euclidean signature. The matrix-valued $SU_L(2)$ isospin gauge field is

$$\widehat{W}_\mu \equiv W^a_\mu \tau^a = \vec{W}_\mu \cdot \vec{\tau}$$

with τ^a the isospin Pauli matrices, Y_μ is the (Abelian) $U_Y(1)$ hypergauge field, and

$$\vec{G}_{\mu\nu}(W) = \partial_\mu \vec{W}_\nu - \partial_\nu \vec{W}_\mu - g\,\vec{W}_\mu \times \vec{W}_\nu\,, \tag{3.2}$$

$$F_{\mu\nu}(Y) = \partial_\mu Y_\nu - \partial_\nu Y_\mu\,. \tag{3.3}$$

The $SU_L(2)\times U_Y(1)$ covariant derivative is

$$D_\mu = \mathbb{1}\,\partial_\mu - i\frac{g}{2}\widehat{W}_\mu - i\frac{g'}{2}Y_\mu\,\mathbb{1}\,, \tag{3.4}$$

where $\mathbb{1}$ is the 2×2 unit matrix in the isospin space. The complex isospinor Higgs field Φ is decomposed as follows,

$$\Phi\ = \phi\,\mathcal{X} \quad \text{with} \quad \phi = \rho\, e^{i\theta} \quad \& \quad \mathcal{X} = \ \mathcal{U}\begin{pmatrix}0\\1\end{pmatrix}. \tag{3.5}$$

Here ϕ is a complex field, $\mathcal{X}$ a two-component complex isospinor, and $\mathcal{U}$ a $SU_L(2)$ matrix. The

$$G_{WS} = SU_L(2)\times U_Y(1)$$

gauge transformation acts on Φ as follows,

$$\Phi \stackrel{G_{\rm WS}}{\longrightarrow} e^{i\omega_Y}\Omega\,\Phi \quad \Rightarrow \quad \begin{cases}\phi & \longrightarrow e^{i\omega_Y}\phi\\ \mathcal{X} & \longrightarrow \Omega\mathcal{X}\end{cases} \tag{3.6}$$

where $\Omega \in SU_L(2)$ and $e^{i\omega_Y} \in U_Y(1)$. As a consequence the decomposition separates isospin from hypercharge [9]. It also introduces a new (internal) *compact* gauge group

$$U_{\text{int}}(1): \quad \begin{array}{l} \phi \to e^{-i\omega_c}\phi \\ \mathcal{X} \to e^{i\omega_c}\mathcal{X} \end{array} \tag{3.7}$$

which leaves the field Φ intact. The spinor $\mathcal{X} \equiv \mathcal{X}_1$ and its isospin conjugate

$$\mathcal{X}_2 = e^{i\beta} i\tau_2 \mathcal{X}^*$$

form an orthonormal basis ($i, j = 1, 2$ and $a, b = \uparrow, \downarrow$),

$$\mathcal{X}_i^\dagger \cdot \mathcal{X}_j \equiv \sum_{a=\uparrow,\downarrow} \mathcal{X}_{ia}^* \mathcal{X}_{aj} = \delta_{ij}$$

$$\sum_{i=1,2} \mathcal{X}_{ia} \mathcal{X}_{ib}^\dagger = \delta_{ab}$$

Hereafter we set $\beta = 0$ as it parameterizes an internal degree of freedom that was already accounted for by (3.7).

We introduce the non-Abelian supercurrents in parallel with Eq. (2.1). For this we expand the covariant derivative of the Higgs field in the spinor basis $(\mathcal{X}_1, \mathcal{X}_2)$,

$$D_\mu \Phi = \left[\frac{1}{\rho}\partial_\mu \rho + \frac{i}{2}\Big(g J_\mu^3 - g' \mathcal{Y}_\mu\Big)\right] \Phi - i\frac{g}{2} J_\mu^+ \cdot \Phi_c \tag{3.8}$$

Here

$$\Phi_c = \phi \mathcal{X}_2$$

is the isocharge conjugated Higgs field. The supercurrents J_μ^+, J_μ^3 and $\mathcal{Y}_\mu$ emerge when we project out the spinor components

$$J_\mu^+ = \frac{2i}{g}\mathcal{X}_2^\dagger \left(\partial_\mu - \frac{ig}{2}\widehat{W}_\mu\right)\mathcal{X}_1 \equiv \vec{W}_\mu \cdot \vec{e}_+ + \frac{i}{g}\vec{e}_3 \cdot \partial_\mu \vec{e}_+ \,, \tag{3.9}$$

$$J_\mu^3 = \frac{2}{ig}\mathcal{X}_1^\dagger \left(\partial_\mu - \frac{ig}{2}\widehat{W}_\mu\right)\mathcal{X}_1 \equiv \vec{W}_\mu \cdot \vec{e}_3 - \frac{i}{2g}\vec{e}_- \cdot \partial_\mu \vec{e}_+ \,, \tag{3.10}$$

$$\mathcal{Y}_\mu = \frac{i}{g'|\phi|^2}\left[\phi^\star \Big(\partial_\mu - i\frac{g'}{2}Y_\mu\Big)\phi - c.c.\right]. \tag{3.11}$$

Here

$$J_\mu^+ = J_\mu^1 + i J_\mu^2$$

and

$$\vec{e}_+ \equiv \vec{e}_1 + i\vec{e}_2$$

with $\vec{e}_i$ $(i = 1, 2, 3)$ three mutually orthogonal unit vectors,

$$\vec{e}_3 = -\frac{\Phi^\dagger \hat{\vec{\tau}} \Phi}{\Phi^\dagger \Phi} \equiv -\mathcal{X}_1^\dagger \hat{\vec{\tau}} \mathcal{X}_1 \tag{3.12}$$

$$\vec{e}_+ = \mathcal{X}_2^\dagger \hat{\vec{\tau}} \mathcal{X}_1 \tag{3.13}$$

and the internal gauge group (3.7) acts as follows,

$$U_{\rm int}(1): \qquad \begin{array}{c} \vec{e}_3 \to \vec{e}_3 \\ \vec{e}_+ \to e^{2i\omega_c}\vec{e}_+ \end{array} \tag{3.14}$$

In parallel with the Abelian Higgs model we interpret (3.9)-(3.11) as a change of variables from the original twenty (real) fields (W^a_μ, Y_μ, Φ) to a new set of twenty fields. In addition of the sixteen $(J^a_\mu, \mathcal{Y}_\mu)$ these include the modulus ρ and the orthogonal triplet $\vec{e}_i$.

The supercurrents $(J^3_\mu, J^+_\mu, \mathcal{Y}_\mu)$ are the manifestly $G_{\rm WS} = SU_L(2) \times U_Y(1)$ gauge invariant *electroweak supercurrents*. They are the non-Abelian generalizations of (2.1). But under the internal gauge symmetry (3.7) supercurrents $(J^3_\mu, J^+_\mu, \mathcal{Y}_\mu)$ transform nontrivially, in the following manner:

$$U_{\rm int}(1): \qquad \begin{array}{rcl} J^+_\mu & \to & e^{2i\omega_c} J^+_\mu\,, \\ J^3_\mu & \to & J^3_\mu + \frac{2}{g}\partial_\mu\omega_c\,, \\ \mathcal{Y}_\mu & \to & \mathcal{Y}_\mu + \frac{2}{g'}\partial_\mu\omega_c\,. \end{array} \tag{3.15}$$

Finally, we note that quite similar electroweak supercurrents have been previously presented in [10]. See also [11] for a related construction.

4. Electroweak Lagrangian in Supercurrent Variables

In terms of the $G_{WS} = SU_L(2) \times U_Y(1)$ invariant variables the classical electroweak Lagrangian (3.1) acquires a form similar to (2.3),

$$\begin{aligned} \mathcal{L}_{\rm WS} = \frac{1}{4}\left(\vec{G}_{\mu\nu}(\vec{J}) + \frac{4\pi}{g}\vec{\widetilde{\Sigma}}_{\mu\nu}\right)^2 + \frac{1}{4}\left(F_{\mu\nu}(\mathcal{Y}) + \frac{4\pi}{g'}\widetilde{\sigma}^\phi_{\mu\nu}\right)^2 \\ +(\partial_\mu\rho)^2 + \frac{\rho^2}{4}\left(gJ^3_\mu - g'\mathcal{Y}_\mu\right)^2 + \frac{\rho^2 g^2}{4} J^+_\mu J^-_\mu + \lambda\,\rho^4 + \mu^2\rho^2 \end{aligned} \tag{4.1}$$

Here $\vec{G}_{\mu\nu}$ and $F_{\mu\nu}$ are now the curvatures of $\vec{J}_\mu$ *resp.* $\mathcal{Y}_\mu$,

$$\vec{G}_{\mu\nu}(\vec{J}) = \partial_\mu\vec{J}_\nu - \partial_\nu\vec{J}_\mu - g\,\vec{J}_\mu \times \vec{J}_\nu\,, \tag{4.2}$$

$$F_{\mu\nu}(\mathcal{Y}) = \partial_\mu\mathcal{Y}_\nu - \partial_\nu\mathcal{Y}_\mu\,. \tag{4.3}$$

The $G_{\rm WS}$–invariant dual string tensor

$$\widetilde{\sigma}^\phi_{\mu\nu} = \frac{1}{2\pi}\,[\partial_\mu, \partial_\nu]\,\arg\phi \tag{4.4}$$

describes the embedding of (singular) stringlike vortex cores in (4.1) in analogy with (2.4). Its non-Abelian and $G_{\rm WS}$–invariant extension generalizes Eq. (4.4) to $SU_L(2)$,

$$\widetilde{\Sigma}^i_{\mu\nu} = \frac{i}{\pi}{\rm Tr}\left[\hat{\tau}^i\Big(\mathcal{U}^\dagger[\partial_\mu,\partial_\mu]\mathcal{U}\Big)\right] \equiv -\frac{1}{8\pi}\epsilon^{ijk}\big(\vec{e}^{\,j}\cdot[\partial_\mu,\partial_\nu]\,\vec{e}^{\,k}\big) \tag{4.5}$$

The (singular) codimension two surfaces described by (4.5) in $\mathbb{R}^4$ are world lines of stringlike vortex cores. The boundaries of these surfaces are curves in $\mathbb{R}^4$ that describe the world lines of pointlike structures including the cores of Wu-Yang type magnetic monopoles.

Note that in (4.1) the orthogonal triplet $\vec{e}_i$ only appears thru the topological structures that are described by the tensors (4.4) and (4.5). This leaves us with seventeen regular and manifestly $SU_L(2) \times U_Y(1)$ gauge invariant field variables: The original gauge symmetry has been entirely eliminated by the change of variables and without any gauge fixing. The only surviving local gauge invariance of (4.1) is the novel internal $U_{\rm int}(1)$ gauge symmetry (3.15), that now defines the Maxwellian gauge symmetry of (4.1). In particular, since there are no local remaining gauge symmetries to be broken there are no issues with Elitzur's theorem. Furthermore, since $\rho \geq 0$ there are no discrete symmetries to be broken by the *v.e.v.* of ρ. But if the minimum of the potential occurs at $\rho = 0$ the supercurrents are massless, and if the minimum occurs at $\rho \neq 0$ three of the supercurrents acquire a nonvanishing mass.

Note that if one overlooks topological structures the functional form of the Lagrangian (4.1) coincides with that of the original Lagrangian in the unitary gauge, even though here no gauge fixing has taken place.

The $G_{\rm WS}$–gauge invariant W–bosons are $W_\mu^\pm = J_\mu^\pm$, and Z–boson and photon A_μ are

$$Z_\mu = \cos\theta_W \, J_\mu^3 - \sin\theta_W \, \mathcal{Y}_\mu \,, \tag{4.6}$$

$$A_\mu = \sin\theta_W \, J_\mu^3 + \cos\theta_W \, \mathcal{Y}_\mu \,, \tag{4.7}$$

where θ_W is the Weinberg angle,

$$\sin\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}$$

Under the internal $U_{\rm int}(1)$ gauge symmetry (3.15) the W–boson field transforms as a charged vector field. From (3.15) and (4.7) we conclude that the Z-boson is a singlet while for the photon we get

$$U_{\rm int}(1): \qquad A_\mu \to A_\mu + \frac{2}{g'g}\sqrt{g^2 + g'^2}\, \partial_\mu \, \omega_c \,. \tag{4.8}$$

In the case of a singular ω_c this gauge transformation acts on the Abelian field strength tensor as follows,

$$F_{\mu\nu} \equiv \; \partial_{[\mu,} A_{\nu]} \to F_{\mu\nu} + \frac{4\pi}{g'g} \cdot n\sqrt{g^2 + g'^2}\, \widetilde{\sigma}_{\mu\nu}^{\rm Dirac} \,, \tag{4.9}$$

where

$$\widetilde{\sigma}^{\rm Dirac} = \frac{1}{2\pi}[\partial_\mu, \partial_\nu]\, \omega_c$$

The location of the (singular) Dirac worldsheet describes an oriented two-dimensional manifold (in $\mathbb{R}^4$) at which the transformation function ω_c has a singularity. Since the string (in $\mathbb{R}^3$) must be unobservable we arrive at the quantization of the worldsheet pre-factor in (4.9) in terms of elementary magnetic charge $4\pi/e$. In this manner the compactness of the internal group provides us with the familiar identification of the electric charge,

$$e = g \sin\theta_W \,.$$

As an example, the Higgs field of a static Nambu monopole [12] is

$$\Phi = \eta f(r) \begin{pmatrix} \cos\theta/2 \\ e^{i\varphi}\sin\theta/2 \end{pmatrix} \tag{4.10}$$

where (r, θ, φ) are spherical coordinates. The singular structures (4.4), (4.5) are $\sigma^{\phi}_{\mu\nu} = 0$, $\Sigma^{+}_{\mu\nu} = 0$, and the string

$$\Sigma^{3}_{\mu\nu} = \delta(x_1)\delta(x_2)\theta(x_3)[\delta_{\mu 3}\delta_{\nu 4} - \delta_{\mu 4}\delta_{\nu 3}] \tag{4.11}$$

ends at the world line of the monopole, located at r=0. The asymptotic behavior of the monopole's fields [12],

$$g\vec{W}_{\mu} = -\vec{e}_3 \times \partial_{\mu}\vec{e}_3 - \vec{e}_3 \cos^2\theta_W \xi_{\mu} \,, \tag{4.12}$$

$$g' Y_{\mu} = \sin^2\theta_W \xi_{\mu} \,, \tag{4.13}$$

together with (4.10) yields for the supercurrents (3.9), (3.10) and (3.11) the asymptotic behaviours

$$J^{+}_{\mu} = 0 \tag{4.14}$$

$$g J^{3}_{\mu} = g' \mathcal{Y}_{\mu} = \sin^2\theta_W \, \xi_{\mu} \tag{4.15}$$

so that asymptotically

$$\begin{aligned} Z_{\mu} &= 0 \\ A_{\mu} &= \tfrac{\sin^2\theta_W}{e}\xi_{\mu} \,, \\ \xi_{\mu} &= -i(\chi_1^{\dagger}\partial_{\mu}\chi_1 - \partial_{\mu}\chi_1^{\dagger}\chi_1) = (1-\cos\theta)\partial_{\mu}\varphi \end{aligned}$$

where ξ_{μ} is the conventional field of the Dirac monopole. The singularity structures show that the monopole possess the non-Abelian charge $4\pi/g$ while the magnetic hypercharge (i.e. the charge with respect to the Y-field) is identically zero, consistent with known results [13].

In our variables, the gauge invariant 't Hooft tensor [14] can be written as

$$G_{\mu\nu} \equiv \vec{G}_{\mu\nu} \cdot \vec{e}_3 - \frac{1}{g}(\vec{e}_3 \cdot D_{\mu}\vec{e}_3 \times D_{\nu}\vec{e}_3) = \partial_{[\mu,} J^{3}_{\nu]} \tag{4.16}$$

where D_{μ} is the $SU_L(2)$ covariant derivative (3.4). The 't Hooft tensor relates to the current j^{N}_{μ} that describes the world trajectory $\mathcal{C}$ of the Nambu monopole,

$$\partial_{\nu}\tilde{G}_{\mu\nu} = \frac{4\pi}{g} j^{N}_{\mu} \;\equiv\; \frac{4\pi}{g} \cdot \int_{\mathcal{C}} \mathrm{d}y_{\mu}\, \delta^{(4)}(x-y) \,, \tag{4.17}$$

The electroweak model also possesses various string solutions [13] including Z–vortices and W–vortices [15, 16] and superconducting strings [17]. For example, the Z-string [15, 13] has a singularity only in the Abelian tensor (4.4): $\vec{\Sigma}_{\mu\nu} = 0$ and

$$\sigma^{\phi}_{\mu\nu} = \delta(x_1)\delta(x_2)[\delta_{\mu 3}\delta_{\nu 4} - \delta_{\mu 4}\delta_{\nu 3}] \,, \tag{4.18}$$

while the W-string [16, 13] leads to $\Sigma^3_{\mu\nu} = 0$, $\sigma^\phi_{\mu\nu} = 0$ and

$$\Sigma^+_{\mu\nu} = e^{i\gamma}\delta(x_1)\delta(x_2)[\delta_{\mu 3}\delta_{\nu 4} - \delta_{\mu 4}\delta_{\nu 3}]\,, \tag{4.19}$$

where γ is a phase.

Finally, we note that a representation of electroweak Lagrangian in terms of gauge invariant variables has also been considered in [18]. But the approach introduced there is quite different from the present one. We also draw attention to the strong coupling interpretation of electroweak Lagrangian proposed in [19].

5. Conformal Geometry

We now propose the Lagrangian (4.1) a generally covariant interpretation. For this we analytically continue to Minkowski space with signature $(-+++)$ and interpret ρ^2 in (4.1) as a dilaton *i.e.* as the conformal scale of a locally conformally flat metric tensor [6],

$$\mathcal{G}_{\mu\nu} \;=\; \left(\frac{\rho}{\kappa}\right)^2 \eta_{\mu\nu} \tag{5.1}$$

Here $\eta_{\mu\nu}$ is the flat Minkowski metric. Since ρ has the dimensions of mass, we introduce the *a priori* arbitrary mass parameter κ to ensure that the metric tensor has the correct dimensionality.

We accept the prescription in [20], to analytically continue the conformal scale (in the case of asymptotically Euclidean manifolds [20]) according to $\rho = 1 + \xi \to 1 - i\xi$, when identifying the Minkowski signature metric tensor.[1] We then conclude that the Minkowski signature Lagrangian (4.1) can be given the following manifestly generally covariant interpretation,

$$\mathcal{L}_{\rm WS} \;=\; \sqrt{-\mathcal{G}}\left\{\frac{1}{16\pi G}(R - 2\Lambda) + \mathcal{L}_M\right\} \tag{5.2}$$

with the matter Lagrangian $\mathcal{L}_M$

$$\mathcal{L}_M = -\frac{1}{4}\mathcal{G}^{\mu\rho}\mathcal{G}^{\nu\sigma}\vec{G}_{\mu\nu}\vec{G}_{\rho\sigma} - \frac{1}{4}\mathcal{G}^{\mu\rho}\mathcal{G}^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma} - \kappa^2(g^2 + g'^2)\mathcal{G}^{\mu\nu}Z_\mu Z_\nu - \kappa^2 g^2\mathcal{G}^{\mu\nu}W^+_\mu W^-_\nu \tag{5.3}$$

We have here introduced parameters $G = 3/(8\pi\kappa^2)$ and $\Lambda = (9\lambda)/(8\pi G)$ and for simplicity of notation the tensors $\vec{G}_{\mu\nu}$ and $F_{\mu\nu}$ now contain (4.5) and (4.4) respectively.

The result (5.2), (5.3) re-interprets the electroweak theory as a generally covariant gravity theory with massive vector fields Z and $W^\pm$ and the (massless) photon A_μ.

Note that in (5.3) we have removed the (bare) Higgs mass term that is present in (3.1), as the Higgs mass term is no longer neeeded in order for the theory to acquire its desired physical properties. In terms of the present variables the correctly normalized masses for Z and $W^\pm$ are provided by the couplings g and g' and the parameter κ, with no reference to the structure of the Higgs potential and/or the mass of the Higgs. We also note that the

[1] We note that the prescription has thus far been properly justified only in the case of pure Einstein action.

location of structures such as vortex and monopole cores where $\rho = 0$ can be interpreted in terms of spacetime singularities.

We are now in a position to analyze the ground state structure of the electroweak theory in the present variables. For this we first note that in the ground state the massive vector fields Z_μ and $W_\mu^\pm$ and the photon field A_μ all must vanish. Consequently the ground state is determined by minimizing the gravitational contribution (5.2). This leads us to the de Sitter metric in its original form,

$$ds^2 \;=\; \left(\frac{\rho^2}{\kappa^2}\right)\eta_{\mu\nu}dx^\mu dx^\nu \;=\; \frac{\eta_{\mu\nu}dx^\mu dx^\nu}{\left[1+\frac{4\pi}{9}G\Lambda\cdot x^2\right]^2}$$

where the conformal scale is a solution to the following equation of motion of "$\lambda\phi^4$" theory,

$$-\Box\left(\frac{\rho}{\kappa}\right) - 8\cdot\frac{4\pi}{9}G\Lambda\left(\frac{\rho}{\kappa}\right)^3 = 0$$

As a consequence in the present variables the ground state of the electroweak theory is the de Sitter space.

If we recall that a four dimensional $\lambda\phi^4$ scalar field theory is trivial [21] and adapt this result to the present case, we conclude that the "cosmological constant" $\Lambda \to 0$. In this limit we recover the flat Minkowski space and the ground state value of ρ coincides with the parameter κ. Thus, in this limit we arrive at the conventional symmetry breaking picture of the original Weinberg-Salam model.

We also comment that if we do not follow the prescription in [20] the gravity Lagrangian acquires the form

$$\mathcal{L}_{gravity} \;=\; \sqrt{-\mathcal{G}}\frac{1}{16\pi G}(-R-2\Lambda)$$

This leads to a wrong sign in the Einstein equation in the presence of matter fields. Now the ground state is anti-de Sitter space, and when we employ stereographically projected coordinates we find

$$ds^2 \;=\; \left(\frac{\rho^2}{\kappa^2}\right)\eta_{\mu\nu}dx^\mu dx^\nu \;=\; \frac{\eta_{\mu\nu}dx^\mu dx^\nu}{\left[1-\frac{4\pi}{9}G\Lambda\cdot x^2\right]^2}$$

where the conformal scale emerges as a solution to the equation of motion of the following "$\lambda\phi^4$" equation of motion,

$$-\Box\left(\frac{\rho}{\kappa}\right) + 8\cdot\frac{4\pi}{9}G\Lambda\left(\frac{\rho}{\kappa}\right)^3 = 0$$

We conclude this Section with the following comments: The derivation of (5.2), (5.3) employs the separation between isospin and hypercharge. In parallel with spin-charge separation in strongly correlated electron systems [4]-[6] it becomes natural to interpret (5.2), (5.3) as a description of the electroweak theory in a strongly coupled/strongly correlated (material) regime. From the point of view of the original electroweak theory, the Lagrangian (5.2), (5.3) should now be interpreted in terms of an effective Lagrangian which has been computed in the strongly coupled/strongly correlated regime using the covariant background field formalism. The full effective Lagrangian accounts for all quantum fluctuations in the fields of the original tree-level electroweak Lagrangian (3.1). But since its explicit form is not available beyond a few leading terms in a loop expansion, we have to resort to an indirect analysis: By employing general arguments of gauge invariance we expect that in terms of the original variables the full effective Lagrangian is a functional of the background fields and their background covariant derivatives. In the low momentum infrared limit where we can ignore the higher order derivative contributions, and since the full result is unknown to us, we may for simplicity proceed by considering the infrared limit in its lowest order. This limit coincides with (5.2), (5.3). After all, the original classical Lagrangian *should* be an important ingredient of the full quantum Lagrangian!

On the other hand, from the point of view of duality arguments [7] it becomes attractive to view the gravity Lagrangian (5.2), (5.3) as a weak field and short distance limit of a more complete gravity theory. For example, the locally conformally flat form of the metric tensor (5.1) could be interpreted as the short distance limit that emerges from a (renormalizable) higher derivative gravity theory with a Lagrangian that contains the following terms,

$$\mathcal{L}_W \ = \ \frac{\sqrt{-\mathcal{G}}}{16\pi G}\,(R-2\Lambda) \ + \ \sqrt{-\mathcal{G}}\cdot\gamma\, W^2_{\mu\nu\rho\sigma}$$

Here $W^2_{\mu\nu\rho\sigma}$ is the Weyl tensor. In the short distance limit the one-loop β-function for γ sends this coupling to infinity [22]. This enforces asymptotically at short distances the condition

$$W_{\mu\nu\rho\sigma} \ \sim \ 0$$

which implies that locally, and in the absence of space-time singularities, the short-distance metric tensor in this more complete gravity theory assumes the conformally flat form (5.1).

6. Summary

In summary, we have shown that in the Weinberg-Salam model the $SU_L(2)\times U_Y(1)$ gauge dependence can be completely eliminated by a mere change of variables and without any gauge fixing. As a consequence issues related to Elitzur's theorem become obsolete. The ensuing Lagrangian describes the electromagnetic interactions of the gauge invariant and massive W and Z bosons. Furthermore, when we interpret the Higgs field as a dilaton in a locally conformally flat spacetime, the electroweak Lagrangian acquires a generally covariant form and the vector bosons receive their correct masses with no reference to any symmetry breaking by a Higgs potential. Moreover, the ground state can be interpreted as the four dimensional de Sitter space. However, this interpretation assumes that we adopt the description of [20]. Otherwise, the ensuing Einstein equation has a wrong sign for the matter coupling, and the gravity interaction becomes repulsive with anti-de Sitter space as the ground state of the theory.

We hope that our manifestly gauge invariant formulation of the electroweak Lagrangian becomes valuable in properly interpreting the structure of the electroweak transition which is soon to be revealed at LHC.

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