*Ludvig D. Faddeev*

# A Mathematician's View of the Development of Physics*

Mathematics in its clean form is the product of the free human mind.

Physics is a natural science with just a single goal – uncovering the structure of matter.

In their quest physicists naturally use mathematical tools to correlate data, to express the laws found by means of formulas and to make relevant calculations. To a greater or smaller extent this is done in all sciences. And there is no *a priori* reason for such a distinguished role of mathematics in physics which we are witnessing nowadays – namely that of an imminent language of physical theory.

I shall not elaborate on the examples to prove this role. Everybody in his profession can choose his favorite. It is enough to recall that such purely mathematical structures as Riemann geometry or Lie groups theory are indispensable in modern theory of gravity, formulated by Einstein, or in the description of kinematical and dynamical symmetries of any physical system.

This role of mathematics as the language of physics is taken by physicists with mixed feelings of admiration and irritation. Take for example the title of the famous essay by Wigner – "On the unreasonable effectiveness of mathematics in natural sciences". The complex formed by these feelings is sometimes resolved in malicious jokes on mathematics which some great men allowed themselves to tell. I shall not comment more on this.

Instead, I shall take seriously the stated role of mathematics as a fact and try to present in this spirit the analysis of modern trends in physics. To do this I shall need some formalized framework and I proceed now to its description.

In the description of the physical system we use two main notions: those of observables and those of states. The set of observ-

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ables $\mathfrak{A}$ comprises all physical entities $A$, $B$, $C$, ... constituting the system. The set of states $\Omega$, with elements $\omega$, $\mu$, ..., describes the possible results of the measurements of observables. More formally, each state $\omega$ gives to each observable $A$ its probability distribution – a nonnegative, monotone increasing function $\omega_A(\lambda)$ of a real variable $\lambda$, $-\infty<\lambda<\infty$, normalized by the conditions $\omega_A(-\infty)=0$, $\omega_A(\infty)=1$. In particular, the mean value of an observable $A$ in a state $\omega$ is given by

$$\langle\omega|A\rangle=\int_{-\infty}^{\infty}\lambda\, d\omega_A(\lambda).$$

The completeness of such a description is expressed in the requirement that states separate observables. Namely, if two observables $A$ and $B$ have the same mean value in all states, then they coincide. This is a formal expression of the main epistemological principle of the ability of cognition of the universe.

Mathematically this principle introduces some structure in the set of observables:

1. $\mathfrak{A}$ is a real linear space.

Indeed, observables $A+B$ and $kA$ for real $k$ are defined as having the mean values

$$\langle\omega|A+B\rangle=\langle\omega|A\rangle+\langle\omega|B\rangle$$

and

$$\langle\omega|kA\rangle=k\langle\omega|A\rangle$$

in all states $\omega$.

2. For each real-valued function $\varphi(\lambda)$ of the real-variable $\lambda$ and each observable $A$ we can construct the observable $\varphi(A)$ by means of the formula

$$\langle\omega|\varphi(A)\rangle=\int_{-\infty}^{\infty}\varphi(\lambda)\, d\omega_A(\lambda),$$

valid for any state $\omega$. Alternatively, we can say that the probability distribution of $\varphi(A)$ is given by

$$\omega_{\varphi(A)}(\lambda)=\omega_A(\varphi^{-1}(\lambda)).$$

To introduce the dynamics of the system we are to describe the notion of motions or one-parameter automorphisms $A\to A(s)$ in the set of observables. This is done by means of a binary operation (bracket) $\{A,B\}$ which allows one to associate a particular